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Cosmological Applications of a Geometrical Interpretation of "*c*"

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Abstract We make the hypothesis that the velocity of light and the expansion of the universe are two aspects of one single concept connecting space and time in the expanding universe. We show that solving Friedman's equations with that interpretation (keeping c = constant) could explain number of unnatural features of the standard cosmology. We thus examine in that light the flatness problem, the problem of the observed uniformity in term of temperature and density of the cosmological background radiation and the small-scale inhomogeneity problem. We finally show that using this interpretation of c leads to reconsider the Hubble diagram of distance moduli and redshifts as obtained from recent observations of type Ia supernovae without having to need an accelerating universe.

Keywords Cosmology · Cosmic background radiation · Cosmology: observation · Cosmological parameters · Flatness · Speed of light · Accelerating universe · Small-scale inhomogeneity · Large-scale smoothness

1 Introduction

The constant c was first introduced as the speed of light. However, with the development of physics, it came to be understood as playing a more fundamental role, its significance being not directly that of a usual velocity (even though its dimensions are) and one might thus think of c as being a fundamental constant of the universe. Moreover, the advent of Einsteinian relativity, the fact that c does appear in phenomena where there is neither light nor any motion (for example in the fundamental equation $E = mc^2$) and its double-interpretation in terms of "velocity of light" and of "velocity of gravitation" forces everybody to associate c with the theoretical description of space-time itself rather than that of some of its specific contents; it invites us to connect c to the geometry of the universe. In this paper, we propose to connect c to the expansion of the universe and we show some consequences of this interpretation in cosmology. We thus examine the flatness problem and the large-scale smoothness problem

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in the light of that interpretation of c. In the third part, we also show that these results are compatible with the observation of small-scale inhomogeneity leading to the structures seen by astronomers. We finally show that this model doesn't need to consider an accelerating universe to interpret the Hubble diagram of distance moduli and redshifts as obtained from recent observations of type Ia supernovae.

2 A Geometrical Interpretation of *c*

The universal constant c sets up *a universal relation between space and time*. This relation is expressed in the relativistic invariant ds^2 which gives on the light cone $c = dx/dt$. On the other hand, the expansion of the universe *provides another universal relation between space and time* which also has the physical dimension of a velocity.

These two results cannot express a fortuitous coincidence: on the contrary, as in our previous works [1, 2], we consider as a logical necessity that there are not *two different* universal relations between space and time having the same physical dimension of a velocity. We thus suggest that the velocity of light c and the expansion of the universe are two aspects of one single concept connecting space and time in the expanding universe. Taking this into account, we can write in the simplest possible form (of course, all the following results also holds when taking $c = 1$)

$$c = K\frac{dR(t)}{dt}, \tag{1}$$

where K is a positive constant. It must be noted that here "c" and consequently dR/dt is constant (no acceleration or deceleration). That choice could seem to lead to a model which fails to describe the real universe. In fact, *in the usual cosmology*, a constant velocity of expansion needs either a totally empty universe (which is obviously not the case) or a cosmological constant exactly canceling matter density and pressure *at all times* which is not fulfilled in usual theories. However, it will be shown in Sect. 4 that using (1) in the second Friedmann's equation can lead to other solutions.

In [1, 2] we have suggested that c must be the recessional velocity of our antipode (that is to say the variation with respect to time of half the spatial circumference of the universe at cosmological time t) so that $c = \pi dR(t)/dt$. This choice of c makes it to play the role of π in Euclidean geometry and underlines its geometrical nature. It also makes the recessional velocity of the observer's antipode to be c. We choose this value in what follows but another numerical choices for K would lead to similar results.

Equation (1) leads to a new physical interpretation of the Einstein's constant c which thus appears to be related to the variation of $R(t)$ with time. It shows that c can be defined from the knowledge of the geometry of space-time only, that is from its size and its age. It thus really takes the statute of a true geometrical fundamental magnitude of the universe.

3 Some Cosmological Consequences of that Interpretation of *c*

3.1 The Flatness Problem

Our first aim is to show how (1) can solve the flatness problem. The question has been asked to know how Ω (which is the ratio of the mass/energy density of the universe) has been so highly fine-tuned in the past to give an approximately flat universe. We show that with this hypothesis on c, the universe *must appear to be flat whatever it may be* (spherical or not) so that it is not surprising that it is found to be flat.

Noting that, using $c=$ constant, the two Friedmann equations remain valid. The first one is

$$\left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{R(t)^2} + \frac{\Lambda c^2}{3}, \tag{2}$$

where k is the curvature parameter. $R(t)$ is related to the scale factor $a(t)$ by the relation $R(t) = R_0 a(t)$ where the subscript 0 refers to a quantity evaluated at the present time.

In the case of a *flat* universe ($k = 0$) and *when using the standard model*, (2) reduces to

$$\left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3}. \tag{3}$$

In the case of a *spherical* universe ($k = 1$) and *when using the above interpretation of c* (that is to say when using (1)), (2) becomes (the writing of the last term is not modified in order to be clearer in what follows)

$$\left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{8\pi G\rho}{3} - \pi^2\left(\frac{\dot{R}(t)}{R(t)}\right)^2 + \frac{\Lambda c^2}{3}, \tag{4}$$

and consequently becomes

$$\left(\frac{\dot{R}(t)}{R(t)}\right)^2 = \frac{8\pi G}{3}\frac{\rho}{\pi^2+1} + \frac{\Lambda c^2}{3(\pi^2+1)}. \tag{5}$$

So, (5) obtained *with* $k = 1$ when using the present interpretation of "c" is quite similar to (3) obtained *with* $k = 0$ in the standard model. This shows that when using (1) a spherical universe would *experimentally* appear to be flat but *with a smaller mass than expected* in standard cosmology since the mass density is $\rho/(\pi^2+1)$ in (5) whereas it is ρ in (3). This explains why it appears to be natural to *observe* a flat universe even if the probability for such an universe is nearly null because of instability.

3.2 The Large-Scale Smoothness Problem

Let us now consider the large-scale smoothness problem in the light of (1). As everybody knows, the cosmic background radiation (CBR) is amazingly uniform across regions in opposite directions. When looking, for example, at two distant regions on opposite sides of our universe, the observed temperature only fluctuates to about one part per 100. Equation (1) radically changes the causal structure of the universe and explains simply this observation. In fact, it shows that it is the *same* small part of the past universe that is observed in all the possible directions of space so that it is no more surprising to find such a thermal and a density uniformity.

Using both the Robertson-Walker metric and (1) with $K = \pi$ (but, note again that another numerical value of K can be taken without changing essential results), the horizon is obtained by calculating

$$\psi_h = \int_{t_h}^{t_0} c\frac{dt}{R(t)} = \pi\int_{R(t_h)}^{R(t_0)} \frac{dR(t)}{R(t)} = \pi\ln\left(\frac{R(t_0)}{R(t_h)}\right). \tag{6}$$

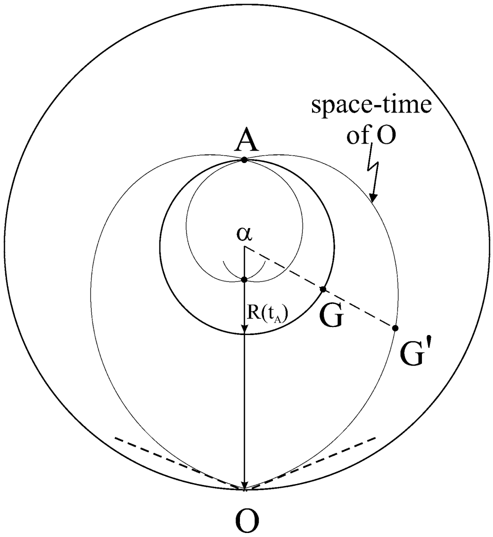

Fig. 1 The space-time of an observer O when using (1). α corresponds to the big-bang. Cosmological time corresponds to the radius line α–O. Space (for each given cosmological time t) corresponds to circles. The space-time of an observer O is constructed by drawing the light cone from O by going back into time and by taking into account the connection of c with the expansion of the universe. It can be seen that, as one goes back in time, the light cone area spreads out to greater distance, and then decreases toward the big-bang. Near O, it corresponds to the usual light cone (*dashed lines*). Globally, it is closed on events A, A', ... which can be observed *in any direction around us*. Among them, A can be identified to the source of the cosmic background radiation. Since it is the same very small volume A of the early universe which is observed in any directions, we have not to be surprised to measure a very high isotropy and homogeneity of the cosmic background radiation (CBR). The universe, at the time t_A that the CBR was emitted, is illustrated by the circle the radius of which is $R(t_A)$. A thus corresponds to only a very small element of the universe at time t_A. The observed isotropy is consequently not contradictory with the existence at this same time t_A of inhomogeneities (such as G). However, such inhomogeneities cannot be seen at time t_A (since only A belongs to the space-time of the observer O at that time) but only at G' at later time $t_{G'}$

In the present model, there exists values of $R(t_h)$ giving $\psi_h > \pi$ so that there is no object horizon. This can be seen by noting that when using (1), all the observed lines of sight of an observer cut themselves in the past so that *there exists an event which can be seen in any directions around us*. This can be computed by using $ds^2 = 0$ along a light ray. It gives the space-time of a given present observer shown in Fig. 1. Solving

$$-c^2dt^2 + R(t)^2\left(\frac{dr^2}{1-kr^2}\right) \equiv -c^2dt^2 + R(t)^2 d\psi^2 = 0, \tag{7}$$

with (1), gives

$$R(t) = R(t_0)\exp\left(-\frac{\psi}{\pi}\right). \tag{8}$$

As a consequence of (8), (1) makes the space-time of a given present observer to be closed on itself at an early time: for $\psi = \pi$, there is an "isotropic event", that is denoted A in Fig. 1, which can be seen in any direction around us and which can be identified to the source of the cosmic background radiation. Since it is thus the *same* event A of the early universe that is seen in any directions (the cosmic microwave background radiation arriving at the earth from all directions in the sky does come from the *same* small part of the early universe), it is not surprising to observe a very high uniformity in terms of temperature and of density of the CBR.

Note that (8) also shows that other "isotropical events" (we mean events which could be observed in any direction around us) could exist. They are defined by $\psi = n\pi$ (where n is an integer). They could correspond to blackbody radiations at temperature other than 2.74 K or to particles other than photons such as cosmic particles. However these other isotropic events can also be non visible because of the opacity of the universe at early times.

3.3 The Small-Scale Inhomogeneity

A related comment concerns the problem of the small-scale inhomogeneity needed to explain the formation of all the observed structures of the universe. Equation (1) shows that the homogeneity of the CBR is compatible with such inhomogenities and consequently with the existence of structures of all scales. In fact, as a consequence of the previous part, the CBR constitutes only a very small part of the past universe and consequently all its other parts (which cannot be seen at the same past age), can be quite different.

Equation (8) with $\psi = \pi$, shows that, at cosmical time t_A, $R(t_A) \neq 0$. Because of this, the source A of the cosmological background radiation in Fig. 1 constitutes only a very small volume element of the universe at that time. The existence of A is consequently consistent with that of spatial inhomogeneities (anywhere else in the universe at the same time t_A that the radiation was emitted). Such inhomogeneities (for example G on the figure), may physically coexist with A at time t_A without being observable at that time (among all the volume elements of the universe at time t_A only the one noted A on Fig. 1 is in the present space-time of the observer O and can consequently be seen). They can be the original seeds giving birth, at later times, to galaxies and other structures which are now observed at $t_{G'} > t_A$ (G' on Fig. 1).

Let us note that this example also shows that it could be possible to see, just behind a galaxy G' (or exactly in the opposite direction), but at an earlier time, the cosmical object which has given birth to that galaxy. Moreover, let us also note that, because of the spiral form of the light rays which are wound round the big-bang, such objects could seem to be older than the value corresponding to the present cosmic time.

3.4 The Hubble Diagram of Distance Moduli and Redshifts

Another problem is the one of the accelerating universe. A very distant supernova with a redshift $z = 1.755$ was recently observed by the Hubble Space Telescope. In the standard cosmology, that observation seems to show that the universe is accelerating. In fact, usual models fail to fit that new data point at redshift 1.755. It can be shown that (1) leads to another expression for the distance-moduli which can fit this point with a good precision (see Fig. 2)

As is well known, the standard expression for the corrected distance-moduli with respect to z can be written [12]

$$\mu = 25 + 5 \log cz - 5 \log H_0 + 1.086(1 - q_0)z + \cdots . \quad (9)$$

Following pioneering works related in [3], recent observations of type Ia supernovae with the Hubble Spatial Telescope [4–7] has provided a robust extension of the Hubble diagram to $1 < z < 1.8$. These new results have shown that observations *cannot be fitted by using* (9)

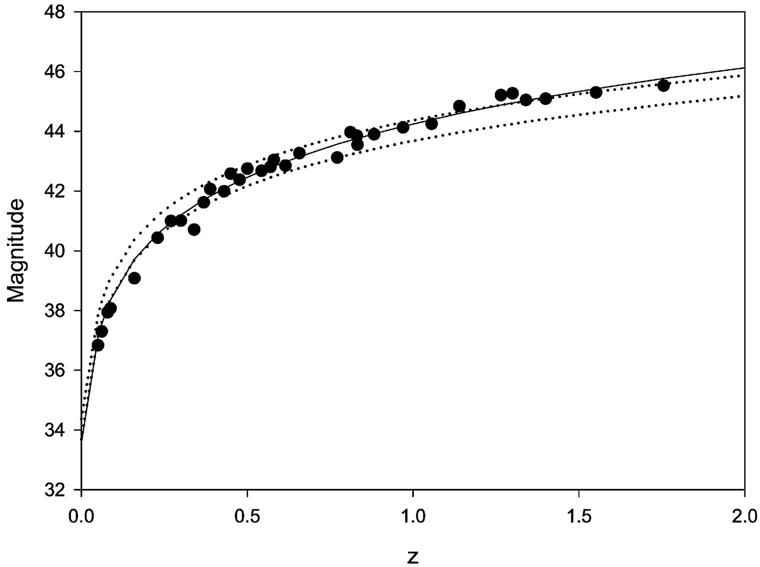

Fig. 2 Hubble diagram (distance modulus vs redshift). The data points are taken from Table 5 of [6]. The two *dashed lines* show the results obtained with (9). One is obtained by fitting the small values of z ($z < 1$). The other is obtained when fitting the high values of z ($z > 1$). They show that it is not possible to fit experimental data *both for* $z < 1$ *and for* $z > 1$ *in the standard model*. The *full line* shows that observations can be fitted both for $z < 1$ and for $z > 1$ by using (10) (and consequently by using (1)). The *full line* which represents predictions of the present model has been drawn by using $H_0 = 58\ \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$

both for $z < 1$ *and for* $z > 1$. To solve this problem, some authors have concluded for the necessity of ruling out the traditional $(\Omega_M, \Omega_\Lambda) = (1, 0)$ universe. The universe would have consequently been decelerating before the current epoch of cosmic acceleration.

When using (1) it can be shown (see the Appendix) that, within the present model, the expression for the corrected distance-moduli is given no more by (9), but by

$$\mu = 25 + 5\log\left(\frac{c}{H(t_0)}\right) + 5\log(1+z) + 5\log\ln(1+z), \tag{10}$$

where c is in km·s^{-1} and H in km·s^{-1}·Mpc^{-1}. As shown on Fig. 2 that result permits to fit the experimental values in the whole range $z < 1$ and for $z > 1$ without any other considerations. Thus, the use of (1) succeeds in fitting all the data without having to consider acceleration.

4 On the General Aspect of the Geometry

Let us now consider the second Friedmann equation:

$$\left(\frac{\ddot{R}(t)}{R(t)}\right) = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right) + \frac{\Lambda c^2}{3}. \tag{11}$$

By using (1) (recalling that $c = \text{constant}$ and neglecting p) that equation leads to

$$\Lambda c^2 \approx 4\pi G\rho. \tag{12}$$

This result can explain why the density of cosmological constant $(\Lambda c^2)/(8\pi G)$ is so near the matter density (it shows why ρ_V is not only small but also, as current Type Ia supernova observations seem to indicate, of the same order of magnitude as the present mass density of the universe). To understand this result note that (12) leads to

$$\Lambda = \frac{4\pi G\rho}{c^2} = \frac{GM}{Rc^2}\frac{3}{R^2}. \tag{13}$$

Introducing (13) with (1) into (5) also gives

$$\Lambda = \frac{\pi^2+1}{\pi^2}\frac{1}{R^2}. \tag{14}$$

Equations (13) and (14) can be solved as

$$\frac{GM}{Rc^2} = \text{Constant}, \tag{15}$$

and

$$\Lambda \approx \frac{1}{R^2}. \tag{16}$$

It is interesting to note that (15) is an expression of the Mach's principle (as for example discussed in [2, 8–10], whereas equation (16) has been shown to be in conformity with quantum cosmology and with existing observations by Chen Wei and Wu Yong-Shi [11]. So, using (1) with c constant can lead to a coherent model (more general solutions could be obtained by using a possible variation of c with respect to time).

5 Conclusions

We propose that "c" is intimately connected to the expansion of the universe. This interpretation gives to c a geometrical meaning and makes of it a true *universal* quantity of the universe which can be defined from its size and its age without any other considerations.

It can solve the problem of flatness: the flat model is unstable; even slight deviations from flatness grow very quickly, leading inevitably, to either a catastrophic big crunch or emptiness. This conflicts with the fact that observations show that the universe is so near flatness. The above interpretation of c solves that problem by showing that the universe *must appear to be flat* (and with a smaller mass than that expected in the standard cosmology) *whatever may be its geometrical form*. It can also give a new light at the horizon problem by explaining the homogeneity and the isotropy of the observed universe at early times and it makes possible the coexistence with the isotropic background radiation of inhomogeneities which can be the seeds of galaxies or of other cosmical objects. It may also permit the observation behind galaxies (or in the opposite direction) of the cosmical object which has given birth to it.

This interpretation of c also leads to a good fitting (without having to consider an accelerating expansion) of observations in Hubble diagram of distance modulus with respect to redshifts as obtained from recent observations of type Ia supernovae.

In concluding, let us underline that usual values obtained from observations in the field of the usual standard cosmology cannot be too quickly transposed to judge the present model. All observations are in fact to be reinterpreted within the present model.

Let us also add that connecting the constant c to the expansion of the universe and more precisely, to the geometry of the universe, may also clarify some intriguing properties of the so-called "speed of light", which appears in phenomena in which neither light nor any motion is present, which also appears in so different fields of physics that are electromagnetism and gravitation and which has both the interpretation of "velocity of light" and of "velocity of gravitation".

Appendix

Let us present the calculations leading to (10). Let an object be at cosmic radial coordinate ψ and consider that the light that is emitted at cosmic time t_e is just reaching us at time t_0. The luminosity distance d_L of the object can be expressed as (see [12])

$$d_L = \left(\frac{R^2(t_o)}{R(t_e)}\right)\psi = R(t_o)(1+z)\psi. \tag{17}$$

Using (1) and noting H_0 the Hubble constant that expression can be written

$$d_L = \left(\frac{c}{\pi H_0}\right)(1+z)\psi, \tag{18}$$

ψ can be obtained by inserting (1) with $K = \pi$ into (7)

$$\psi = \int_{t_e}^{t_o} c\frac{dt}{R(t)} = \pi \ln\left(\frac{R(t_o)}{R(t_e)}\right) = \pi \ln(1+z), \tag{19}$$

so that

$$d_L = \frac{c}{H_0}(1+z)\ln(1+z). \tag{20}$$

Expressing the distance modulus μ in terms of d_L [12] gives

$$\mu = 25 + 5\log(d_L). \tag{21}$$

Inserting then (20) inside (21) thus leads to (10).

References

1. Vigoureux, B., Vigoureux, J.-M., Vigoureux, P.: Phys. Essays **14**(4), 314–319 (2001)
2. Vigoureux, J.-M., Vigoureux, B., Vigoureux, P.: Found. Phys. Lett. **16**(2), 183–193 (2003)
3. Norgaard-Nielsen, H.U., Jorgensen, H.E., Aragon Salamanca, A., Ellis, R.S.: Nature **339**, 523 (1989)
4. Wang, L., Goldhaber, G., Aldering, G., Perlmutter, S.: Astrophys. J. **590**, 944–970 (2003)
5. Tonry, J.L., et al.: Astrophys. J. **594**, 1–24 (2003)
6. Riess, A.G., et al.: Astrophys. J. **607**, 665–687 (2004)
7. Schwarzschild, B.: Phys. Today, 19–21 (2004)
8. Costa de Beauregard, O.: Ann. Fond. Louis de Broglie **23**, 135 (1999)
9. Costa de Beauregard, O.: Found. Phys. Lett. **13**, 395 (2000)
10. Sciama, D.W.: Mon. Not. R. Astron. Soc. **113**, 34 (1953)
11. Chen, W., Wu, Y.-S.: Phys. Rev. D **41**, 695–698 (1990)
12. Weinberg, S.: Gravitation and Cosmology. Wiley, New York (1972)